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Double diffractive $\rho-$production in $\gamma^*\gamma^*$ collisions

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Abstract

We present a first estimate of the cross-section for the exclusive process $\gamma_L^*(Q_1^2)\gamma_L^*(Q_2^2) \to \rho_L^0 \rho_L^0$, which will be studied in the future high energy e^+e^-−linear collider. As a first step, we calculate the Born order approximation of the amplitude for longitudinally polarized virtual photons and mesons, in the kinematical region $s \gg -t,\, Q_1^2\,, Q_2^2$. This process is completely calculable in the hard region $Q_1^2\,, Q_2^2 \gg \Lambda_{QCD}^2$. We perform most of the steps in an analytical way. The resulting cross-section turns out to be large enough for this process to be measurable with foreseen luminosity and energy, for Q_1^2 and Q_2^2 in the range of a few GeV^2.

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1 Introduction

The next generation of e^+e^-−colliders will offer a possibility of clean testing of QCD dynamics. By selecting events in which two vector mesons are produced with large rapidity gap, through scattering of two highly virtual photons, one is getting access to the kinematical regime in which the perturbative approach is justified. If additionally one selects the events with comparable photon virtualities, the perturbative Regge dynamics of QCD of the BFKL [1] type, should dominate with respect to the conventional partonic evolution of DGLAP [2] type. Apart from the study of the total cross section which has been proposed as a test of BFKL dynamics [3, 4], one can achieve similar goals by studying diffractive reactions. From this point of view the production of two J/Ψ−mesons was studied in Ref. [5]. In this case the hard scale is supplied by the mass of the heavy quark. In the present paper we propose to study the electroproduction of two ρ−mesons in the $\gamma^*\gamma^*$ collisions. In this case the virtualities of the scattered photons play the role of the hard scales. As a first step in this direction we shall consider this process with longitudinally polarized photons and ρ−mesons,

$$\gamma^*_L(q_1)\,\gamma^*_L(q_2) \to \rho_L(k_1)\,\rho_L(k_2)\,, \tag{1.1}$$

for arbitrary values of $t = (q_1 - k_1)^2$, with $s \gg -t$. The choice of longitudinal polarizations of both the scattered photons and produced vector mesons is dictated by the fact that this configuration of the lowest twist-2 gives the dominant contribution in the powers of the hard scale Q^2, when $Q_1^2 \sim Q_2^2 \sim Q^2$. The measurable cross section is related to the amplitude of this process through the usual photon flux factors :

$$\frac{d\sigma(e^+e^- \to e^+e^-\rho\rho)}{dy_1 dy_2 dQ_1^2 dQ_2^2} = \frac{1}{Q_1^2 Q_2^2}\frac{\alpha}{2\pi} P_{\gamma/e}(y_1) P_{\gamma/e}(y_2)\sigma(\gamma^*\gamma^* \to \rho\rho)\,, \tag{1.2}$$

where y_i are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the respective leptons and with $P_{\gamma/e}(y) = 2(1-y)/y$ for longitudinally polarized photons. These double tagged events allow to access this hard cross-section [6]. According to our best knowledge this process has not been discussed up to now. At lower energy, some experimental data exist for Q_2^2 small [7] and may be analysed [8] in terms of generalized distribution amplitudes [9] or transition distribution amplitudes [10].

In this paper we calculate the scattering amplitude of the process (1.1) in the Born approximation. In this way we get an estimate of the cross section and prove the feasibility of a dedicated experiment. Partial results have been presented in Ref.[11]. In the near future, we intend to extend this study by taking into account of BFKL evolution and transverse photon polarizations.

2 Kinematics

The process (1.1) at high energies can be visualized as in Fig.1. The physical picture of this impact representation is due to the presence of different time scales: a $q\bar{q}$ dipole is formed from the virtual photon, then interacts by exchanging t−channel gluons before

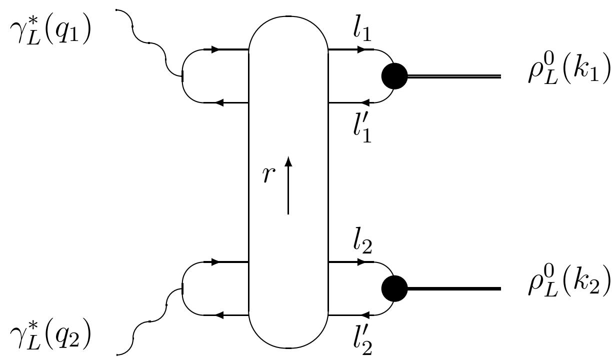

Figure 1: Amplitude for the process $\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0$ in the impact representation. The blobs denote the vector meson distribution amplitudes.

recombining in a meson. The Fig.1 also explains the kinematics of the process (1.1). We parametrize the incoming photon momenta by introducing two light-like Sudakov vectors q_1' and q_2' related to the incoming particles, which satisfy $2q_1' \cdot q_2' = s \equiv 2q_1 \cdot q_2$. The usual $s_{\gamma^*\gamma^*}$ is related to the auxiliary useful variable s by $s_{\gamma^*\gamma^*} = s - Q_1^2 - Q_2^2$. In this basis, the incoming photon momenta read

$$\begin{aligned} q_1 &= q_1' - \frac{Q_1^2}{s} q_2' \,, \\ q_2 &= q_2' - \frac{Q_2^2}{s} q_1' \,. \end{aligned} \tag{2.1}$$

Their polarization vectors are

$$\begin{aligned} \epsilon_\mu^{L(1)} &= \frac{q_{1\mu}}{Q_1} + \frac{2Q_1}{s} q'_{2\mu} \,, \\ \epsilon_\mu^{L(2)} &= \frac{q_{2\mu}}{Q_2} + \frac{2Q_2}{s} q'_{1\mu} \,, \end{aligned} \tag{2.2}$$

which is obtained after imposing the conditions $\epsilon_{L(i)}^2 = 1$ and $q_i \cdot \epsilon_{L(i)} = 0$. Because of e.m. gauge invariance, polarization vectors (2.2) can be effectively replaced by the second terms on the rhs of these equations. The momentum transfer in the $t-$channel is $r = k_1 - q_1$.

We label the momentum of the quarks and antiquarks entering the meson wave functions as l_1 and l_1' for the upper part of the diagram and l_2 and l_2' for the lower part (see fig.1).

In the basis (2.1), the vector meson momenta can be expanded in the form

$$\begin{aligned} k_1 &= \alpha(k_1)\, q_1' + \frac{\underline{r}^2}{\alpha\, s} q_2' + r_\perp \,, \\ k_2 &= \beta(k_1)\, q_2' + \frac{\underline{r}^2}{\beta\, s} q_1' - r_\perp \,. \end{aligned} \tag{2.3}$$

Note that our convention is such that for any tranverse vector $v_\perp$ in Minkowski space, $\underline{v}$ denotes its euclidean form. In the following, we will treat the ρ meson as being massless.

α and β are very close to unity, and read

$$\alpha(k_1) = \frac{1}{2}\left(1 - \frac{Q_2^2}{s}\right)\left(1 + \sqrt{1 - 4\frac{\underline{r}^2}{s}\frac{1}{\left(1-\frac{Q_1^2}{s}\right)\left(1-\frac{Q_2^2}{s}\right)}}\right) ,$$

$$\beta(k_2) = \frac{1}{2}\left(1 - \frac{Q_1^2}{s}\right)\left(1 + \sqrt{1 - 4\frac{\underline{r}^2}{s}\frac{1}{\left(1-\frac{Q_1^2}{s}\right)\left(1-\frac{Q_2^2}{s}\right)}}\right) . \tag{2.4}$$

In this decomposition, it is straightforward to relate $t = r^2$ with $\underline{r}^2 = -r_\perp^2$. The corresponding relation is

$$t = -\frac{Q_1^2 Q_2^2}{s} - \frac{2\underline{r}^2}{1 + \sqrt{1 - \frac{4\underline{r}^2}{s\left(1-\frac{Q_1^2}{s}\right)\left(1-\frac{Q_2^2}{s}\right)}}}\left(\frac{1}{1-\frac{Q_1^2}{s}} + \frac{1}{1-\frac{Q_2^2}{s}} - 1\right) , \tag{2.5}$$

or equivalently,

$$\underline{r}^2 = -\left(t + \frac{Q_1^2 Q_2^2}{s}\right)\frac{\left(1-\frac{Q_1^2}{s}\right)\left(1-\frac{Q_2^2}{s}\right)}{\left(1-\frac{Q_1^2Q_2^2}{s^2}\right)}\left(1 + \frac{t + \frac{Q_1^2Q_2^2}{s}}{s\left(1-\frac{Q_1^2Q_2^2}{s^2}\right)}\right) . \tag{2.6}$$

From Eq.(2.5) the threshold for $|t|$ is given by $|t|_{min} = Q_1^2 Q_2^2/s$, corresponding to $r_\perp = 0$. In the kinematical range we are interested in, the relation (2.6) can be approximated as $\underline{r}^2 = -t$, in accordance with the usual result that r can be considered as purely transverse in the Regge limit.

We write the Sudakov decomposition of the quarks entering the ρ mesons as

$$\begin{aligned}
l_1 &= z_1 q_1' + l_{\perp 1} + z_1 r_\perp - \frac{(l_{\perp 1} + z_1 r_\perp)^2}{z_1 s} q_2' \, , \\
l_1' &= \bar{z}_1 q_1' - l_{\perp 1} + \bar{z}_1 r_\perp - \frac{(-l_{\perp 1} + \bar{z}_1 r_\perp)^2}{\bar{z}_1 s} q_2' \, , \\
l_2 &= z_2 q_2' + l_{\perp 2} - z_2 r_\perp - \frac{(l_{\perp 2} - z_2 r_\perp)^2}{z_2 s} q_1' \, , \\
l_2' &= \bar{z}_2 q_2' - l_{\perp 2} - \bar{z}_2 r_\perp - \frac{(-l_{\perp 2} - \bar{z}_2 r_\perp)^2}{\bar{z}_2 s} q_1' \, ,
\end{aligned} \tag{2.7}$$

where we have explicitly separated the transverse momenta $l_{\perp 1}$ ($l_{\perp 2}$) of quark and anti-quark forming the ρ_L-mesons with respect to the momentum k_1 (k_2). In the following we shall apply the collinear approximation which consists in putting the relative momenta $l_{i\perp}$ in Eq.(2.7) to zero at each $q\bar{q}\rho-$meson vertex. The decomposition (2.7) is more easily understood in a slightly different Sudakov basis, which up to term linear in $r_\perp$ is obtained by the substitutions $q_1' \to q_1' + r_\perp$ and $q_2' \to q_2' - r_\perp$, see Eq.(2.3) with $\alpha(k_1)$ and $\beta(k_2)$ close to unity. In this new basis the ρ mesons have no transverse momenta and quarks transverse momenta have only relative momenta.

3 Impact representation

The impact representation of the scattering amplitude for the process (1.1) has the form (see Fig.2)

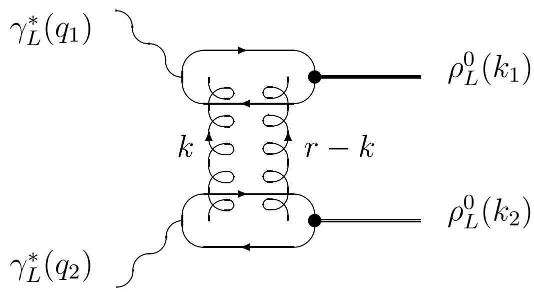

Figure 2: Amplitude for the process $\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0$ at Born order. The t-channel gluons are attached to the quark lines in all possible ways.

$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^4 \underline{k}^2\,(\underline{r}-\underline{k})^2} \mathcal{J}^{\gamma_L^*(q_1)\to\rho_L^0(k_1)}(\underline{k},\underline{r}-\underline{k})\;\mathcal{J}^{\gamma_L^*(q_2)\to\rho_L^0(k_2)}(-\underline{k},-\underline{r}+\underline{k})\,, \quad (3.1)$$

where $\mathcal{J}^{\gamma_L^*(q_1)\to\rho_L^0(k_1)}(\underline{k},\underline{r}-\underline{k})$ ($\mathcal{J}^{\gamma_L^*(q_2)\to\rho_L^0(k_2)}(\underline{k},\underline{r}-\underline{k})$) are the impact factors corresponding to the transition of $\gamma_L^*(q_1) \to \rho_L^0(k_1)$ ($\gamma_L^*(q_2) \to \rho_L^0(k_2)$) via the $t-$channel exchange of two gluons. The impact factors are the $s-$channel discontinuities of the corresponding $S-$matrices describing the $\gamma_L^*\, g \to \rho_L^0\, g$ processes projected on the longitudinal (nonsense) polarizations of the virtual gluons in the $t-$channel

$$\mathcal{J}^{\gamma_L^*(q_1)\to\rho_L^0(k_1)}(\underline{k},\underline{r}-\underline{k}) = \int \frac{d\beta}{s} S_{\mu\nu}^{\gamma_L^*(q_1)\to\rho_L^0(k_1)}\, q_2'^{\mu} q_2'^{\nu}\,, \quad (3.2)$$

$$\mathcal{J}^{\gamma_L^*(q_2)\to\rho_L^0(k_2)}(-\underline{k},-\underline{r}+\underline{k}) = \int \frac{d\alpha}{s} S_{\mu\nu}^{\gamma_L^*(q_2)\to\rho_L^0(k_2)}\, q_1'^{\mu} q_1'^{\nu}\,. \quad (3.3)$$

Here the integration variables α and β are defined by the Sudakov decomposition of the gluonic momentum k

$$k = \alpha q_1' + \beta q_2' + k_\perp\,. \quad (3.4)$$

The amplitude (3.1) depends linearly on s, since these impact factors are s-independent. Calculations of the impact factors in the Born approximation are standard [12]. [3] They read

$$\mathcal{J}^{\gamma_L^*(q_1)\to\rho_L(k_1)}(\underline{k},\underline{r}-\underline{k})$$
$$= \int_0^1 dz_1 z_1\,\bar{z}_1\,\phi(z_1) 8\pi^2\alpha_s \frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c} Q_1\, f_\rho \alpha(k_1)\mathrm{P_P}(\mathrm{z_1},\underline{\mathrm{k}},\underline{\mathrm{r}},\mu_1) \quad (3.5)$$

[3] Recently the forward impact factor of $\gamma_L^*(Q^2) \to \rho_L^0$ transition was calculated at the next-to-leading order accuracy in Ref. [13].

and

$$
\begin{aligned}
&\mathcal{J}^{\gamma_L^*(q_2)\to\rho_L^0(k_2)}(-\underline{k}, -\underline{r}+\underline{k}) \\
&= \int\limits_0^1 dz_2 z_2\, \bar{z}_2\, \phi(z_2) 8\pi^2\alpha_s \frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c} Q_2\, f_\rho \beta(k_2) \mathrm{P_P}(\mathrm{z}_2, \underline{\mathrm{k}}, \underline{\mathrm{r}}, \mu_2)\ ,
\end{aligned}
\tag{3.6}
$$

where

$$
\mathrm{P_P}(\mathrm{z}_1, \underline{\mathrm{k}}, \underline{\mathrm{r}}, \mu_1) = \frac{1}{\mathrm{z}_1^2 \underline{\mathrm{r}}^2 + \mu_1^2} + \frac{1}{\bar{\mathrm{z}}_1^2 \underline{\mathrm{r}}^2 + \mu_1^2} - \frac{1}{(\mathrm{z}_1 \underline{\mathrm{r}} - \underline{\mathrm{k}})^2 + \mu_1^2} - \frac{1}{(\bar{\mathrm{z}}_1 \underline{\mathrm{r}} - \underline{\mathrm{k}})^2 + \mu_1^2} \tag{3.7}
$$

is proportional to the impact factor of quark pair production from a longitudinally polarized photon, with two t-channel exchanged gluons. In formula (3.7) the collinear approximation $l_\perp = 0$ has been made.

In this expression $\mu_1^2 = Q_1^2\, z_1\, \bar{z}_1 + m^2$ and $\mu_2^2 = Q_2^2\, z_2\, \bar{z}_2 + m^2$, where m is the quark mass. The limit $m \to 0$ is regular and we will restrict ourselves to the light quark case, taking thus $m = 0$. Note that Eq.(3.6) can be obtained from Eq.(3.5) from the combination of two substitutions, firstly $(z_1,\, Q_1) \to (z_2,\, Q_2)$ and secondly $(\underline{k},\, \underline{r}) \to (\underline{k} - \underline{r},\, -\underline{r})$. This second substitution is easy to understand since for the upper (lower) blob, the total $t-$channel outgoing momentum is $r_\perp$ $(-r_\perp)$, the outgoing gluon carries momentum k $(k-r)$, and the incoming gluon carries momentum $k-r$ (k). This substitution effectively corresponds to exchanging the third and the fourth term in Eq. (3.7), and leaves this impact factor invariant, due to its symmetry under $z \to \bar{z}$, which is reminiscent of the Pomeron structure of the t-channel state (namely, the impact factor is even under C conjugation). Thus, only the first substitution is necessary to obtain the upper blob from the lower blob.

In the formulae (3.5, 3.6), ϕ is the distribution amplitude of the produced longitudinally polarized $\rho-$mesons. For the case with quark q of one flavour it is defined (see, e.g. [14]) by the matrix element of non-local, gauge invariant correlator of quark fields on the light-cone

$$
\langle 0|\bar{q}(x)\ \gamma^\mu\ q(-x)|\rho_L(p)\rangle = f_\rho\ p^\mu \int\limits_0^1 dz e^{i(2z-1)(px)}\phi(z)\ , \tag{3.8}
$$

where the coupling constant is $f_\rho = 216$ MeV and where the gauge links are omitted to simplify the notation. The amplitudes for production of ρ^0's are obtained by noting that $|\rho^0\rangle = 1/\sqrt{2}(|\bar{u}u\rangle - |\bar{d}d\rangle)$.

The structures in the formulae (3.5, 3.6) arise due to the collinear approximation. Indeed, one can neglect the $l_\perp$ dependencies in the propagators, when computing the impact factors. Thus the only dependency with respect to this transverse momentum of the quark, in the Sudakov basis of the $\rho-$meson, is inside the wave function of these $\rho-$mesons. When integrating over the phase space of the quarks and antiquarks, this results in integrating the meson wave functions up to the factorization scale (which is of the order of the photon virtualities). We neglect here any evolution with respect to this

scale. Such integrated wave function is by definition the distribution amplitude of the $\rho-$mesons. For simplicity, we use the asymptotic distribution amplitude

$$\phi(z) = 6\, z\,(1-z)\,. \tag{3.9}$$

The only remaining part the quark phase space integrations are the integration with respect to quark longitudinal fractions of meson momenta z_1 and z_2.

Combining Eqs.(3.1, 3.5, 3.6, 3.7), the amplitude can be expressed as

$$\mathcal{M} = i\, s\, 2\,\pi\, \frac{N_c^2-1}{N_c^2}\, \alpha_s^2\, \alpha_{em}\alpha(k_1)\,\beta(k_2)\, f_\rho^2\, Q_1\, Q_2 \int_0^1 dz_1\, dz_2\, z_1\, \bar{z}_1\, \phi(z_1)\, z_2\, \bar{z}_2\, \phi(z_2)\mathrm{M}(z_1,\, z_2)\,, \tag{3.10}$$

with

$$\mathrm{M}(z_1,\, z_2) = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{r}-\underline{k})^2} \left[\frac{1}{z_1^2\underline{r}^2+\mu_1^2} + \frac{1}{\bar{z}_1^2\underline{r}^2+\mu_1^2} - \frac{1}{(z_1\underline{r}-\underline{k})^2+\mu_1^2} - \frac{1}{(\bar{z}_1\underline{r}-\underline{k})^2+\mu_1^2}\right]$$
$$\times \left[\frac{1}{z_2^2\underline{r}^2+\mu_2^2} + \frac{1}{\bar{z}_2^2\underline{r}^2+\mu_2^2} - \frac{1}{(z_2\underline{r}-\underline{k})^2+\mu_2^2} - \frac{1}{(\bar{z}_2\underline{r}-\underline{k})^2+\mu_2^2}\right]. \tag{3.11}$$

In term of this amplitude, the differential cross-section can be expressed in the large s limit (neglecting terms of order Q_i^2/s) as

$$\frac{d\sigma^{\gamma_L^*\gamma_L^*\to\rho_L^0\rho_L^0}}{dt} = \frac{|\mathcal{M}|^2}{16\,\pi\, s^2}\,. \tag{3.12}$$

4 Cross section at Born order

4.1 Forward case

We begin with the simpler case $t = t_{min}$ (*i.e.* $\underline{r} = 0$), where the final result for the function $M(z_1, z_2)$ (3.11) can be written in a rather simple form. The integral over $\underline{k}$ can readily be performed and gives

$$M(z_1, z_2) = \frac{4\pi}{z_1\,\bar{z}_1 z_2\,\bar{z}_2 Q_1^2\, Q_2^2\,(z_1\,\bar{z}_1 Q_1^2 - z_2\,\bar{z}_2 Q_2^2)} \ln\frac{z_1\,\bar{z}_1 Q_1^2}{z_2\,\bar{z}_2 Q_2^2}\,. \tag{4.1}$$

The amplitude $\mathcal{M}$ given by Eq.(3.10) can then be computed analytically from Eq.(4.1) through double integration over z_1 and z_2. This is explained in appendix A.1. It results in

$$\mathcal{M}_{t_{min}} = -is\,\frac{N_c^2-1}{N_c^2}\,\alpha_s^2\,\alpha_{em}\,\alpha(k_1)\,\beta(k_2)\,f_\rho^2\,\frac{9\pi^2}{2}\,\frac{1}{Q_1^2Q_2^2}\left[6\left(R+\frac{1}{R}\right)\ln^2 R\right. \tag{4.2}$$
$$+\,12\left(R-\frac{1}{R}\right)\ln R\,+\,12\left(R+\frac{1}{R}\right)\,+\left(3\,R^2+\,2\,+\frac{3}{R^2}\right)\big(\ln\,(1-R)\,\ln^2 R$$
$$\left.-\ln\,(R+1)\,\ln^2 R\,-\,2\,\mathrm{Li}_2\,(-R)\,\ln R\,+\,2\,\mathrm{Li}_2\,(R)\,\ln R\,+\,2\,\mathrm{Li}_3\,(-R)\,-\,2\,\mathrm{Li}_3\,(R)\big)\right]\,,$$

where $R = Q_1/Q_2$.

In the special case where $Q = Q_1 = Q_2$, it simplifies immediately to

$$\mathcal{M}_{t_{\min}}(Q_1 = Q_2) \sim \; -i\, s\, \frac{N_c^2-1}{N_c^2}\, \alpha_s^2\, \alpha_{em}\, \alpha(k_1)\, \beta(k_2)\, f_\rho^2\, \frac{9\pi^2}{2Q^4}\, (24 - 28\, \zeta(3))\,. \tag{4.3}$$

The peculiar limits $R \gg 1$ and $R \ll 1$ are of special physical interest, since they correspond to the kinematics typical for deep inelastic scattering on a photon target described through collinear approximation, *i.e.* the usual parton model. At moderate values of s, apart from diagrams with gluon exchange in $t-$channel considered here, one should also take into account of diagrams with quark exchange. We do not consider them here since we restrict ourselves to the asymptotical region of large s.

In the limit $R \gg 1$, the amplitude simplifies into

$$\mathcal{M}_{t_{\min}} \sim is \frac{N_c^2-1}{N_c^2}\, \alpha_s^2\, \alpha_{em}\, \alpha(k_1)\, \beta(k_2)\, f_\rho^2\, \frac{96\pi^2}{Q_1^2\, Q_2^2} \left(\frac{\ln R}{R} - \frac{1}{6R} \right)\,. \tag{4.4}$$

This result can be obtained directly by imposing from the very beginning the $k_\perp$ ordering typical of parton model. This is shown explicitly in Appendix A.2.

4.2 Non forward case

In this section we will compute the amplitude (3.1), where we have been able to perform analytically the $k_\perp$ integrals. It involves the evaluation of a box diagram with two distinct massive propagators and two massless propagators (denoted $I_{4m_a m_b}$ below). We are not aware of any previous analytic calculation of such an integral. This gives us the possibility of studying various kinematical limits in variables Q_1^2, Q_2^2, t.

$M(z_1,\, z_2)$ as defined in Eq.(3.11) is symmetric under $z_1 \leftrightarrow \bar{z}_1$ and under $z_2 \leftrightarrow \bar{z}_2$. Since on the other hand the distribution amplitude $\phi(z)$ is also symmetric under $z \leftrightarrow \bar{z}$, $M(z_1,\, z_2)$ can be modified by adding any antisymmetric term, since the integration over z_1 and z_2 will automatically select its symmetric part.

One thus writes

$$\mathcal{M} = is 2\pi \frac{N_c^2-1}{N_c} \alpha_s^2 \alpha_{em} \alpha(k_1) \beta(k_2) f_\rho^2 Q_1 Q_2 \int_0^1 dz_1\, dz_2\, z_1\, \bar{z}_1\, \phi(z_1)\, z_2\, \bar{z}_2\, \phi(z_2)\, M_{asym.}(z_1,\, z_2)\;, \tag{4.5}$$

with

$$M_{asym.}(z_1,\, z_2) = \int \frac{d^2\underline{k}}{\underline{k}^2 (\underline{r}-\underline{k})^2} \left[\frac{1}{(z_1^2 \underline{r}^2 + \mu_1^2)(z_2^2 \underline{r}^2 + \mu_2^2)} - \frac{1}{(z_1^2 \underline{r}^2 + \mu_1^2)((z_2 \underline{r} - \underline{k})^2 + \mu_2^2)} \right.$$
$$\left. - \frac{1}{(z_2^2 \underline{r}^2 + \mu_2^2)((z_1 \underline{r} - \underline{k})^2 + \mu_1^2)} + \frac{1}{((z_1 \underline{r} - \underline{k})^2 + \mu_1^2)((z_2 \underline{r} - \underline{k})^2 + \mu_2^2)} \right]\,. \tag{4.6}$$

$M_{asym.}(z_1,\, z_2)$ can be expressed in term of three kind of integrals, namely,

$$I_2 = \int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2}\,, \tag{4.7}$$

$$I_{3m} = \int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2((\underline{k}-\underline{a})^2+m^2)}\,, \tag{4.8}$$

and

$$I_{4m_a m_b} = \int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2((\underline{k}-\underline{a})^2+m_a^2)((\underline{k}-\underline{b})^2+m_b^2)}\,, \tag{4.9}$$

where we use the dimensional regularization $d = 2+2\epsilon$. However, since we will effectively rely, for computing the various integrals involved, on a method which is applicable only for both UV and IR finite integrals, it is more efficient to rewrite $\mathcal{M}$ (3.10) in terms of another asymmetric two dimensional amplitude of the form

$$\begin{aligned}\tilde{M}(z_1,\, z_2) = &-\left(\frac{1}{z_1^2\underline{r}^2+\mu_1^2}+\frac{1}{\bar{z}_1^2\underline{r}^2+\mu_2^2}\right)J_{3\mu_2}(z_2) - (1\leftrightarrow 2)\\ &+J_{4\mu_1\mu_2}(z_1,\, z_2)+J_{4\mu_1\mu_2}(\bar{z}_1,\, z_2)\,,\end{aligned} \tag{4.10}$$

where

$$J_{3\mu}(a) = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\left[\frac{1}{(\underline{k}-\underline{r}a)^2+\mu^2}-\frac{1}{a^2\underline{r}^2+\mu^2}+(a\leftrightarrow \underline{a})\right]\,, \tag{4.11}$$

and

$$\begin{aligned}J_{4\mu_1\mu_2}(z_1,\, z_2) &= \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\\ \times&\left[\frac{1}{((\underline{k}-\underline{r}z_1)^2+\mu_1^2)((\underline{k}-\underline{r}z_2)^2+\mu_2^2)}-\frac{1}{(z_1^2\underline{r}^2+\mu_1^2)(z_2^2\underline{r}^2+\mu_2^2)}+(z\leftrightarrow\bar{z})\right]\,.\end{aligned} \tag{4.12}$$

$J_{3\mu}$ and $J_{4\mu_1\mu_2}$ are two dimensional integrals with respectively 3 propagators (1 massive) and 4 propagators (2 massive, with different masses). They are both IR and UV finite. Their computation by brute force technique, using Feynman parametrization, seems untractable in such a form (specially for $J_{4\mu_1\mu_2}$). However, applying a trick inspired by conformal field theories, it is possible to compute these integrals. The basic idea is to perform special conformal inverse transformations, considered here in momemtum space. Although these two integrals, because of mass terms, are not conformal invariant, this is actually efficient after a suitable redefinition of the massive parameters. This is presented in the appendix A.3.

To complete the evaluation of the amplitude $\mathcal{M}$, one needs to integrate over the quark momentum fractions in the ρ mesons z_1 and z_2. In the general case, for arbitrary values of t, it seems not possible to perform the z_1 and z_2 integrations analytically. We thus do them numerically. In course of them, we observe the absence of end-point singularity when $z_{1(2)} \to 0$ or $z_{1(2)} \to 1$, since P_P as defined in Eq.(3.7) diverges like $1/z$, $1/(1-z)$ when $z \to 0,\, 1$. This leads to perfectly stable numerical integrations.

4.3 Results

We use now the previous formulae in order to get prediction for production rate of diffractive double ρ production. Running of α_S is a subleading effect with respect to our treatment. Anyway, we choose to replace α_S^2 in the various formulae presented above by $\alpha_S(Q_1^2)\,\alpha_S(Q_2^2)$ in order to fix the coupling, and use the three-loop running coupling $\alpha_S(Q_1^2)$ and $\alpha_S(Q_2^2)$ with $\Lambda^{(4)}_{\overline{MS}} = 305\,\mathrm{MeV}$ (see, e.g. [15]).

In Fig. 3, we display the differential cross-section $d\sigma(\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0)/dt$ for vanishing

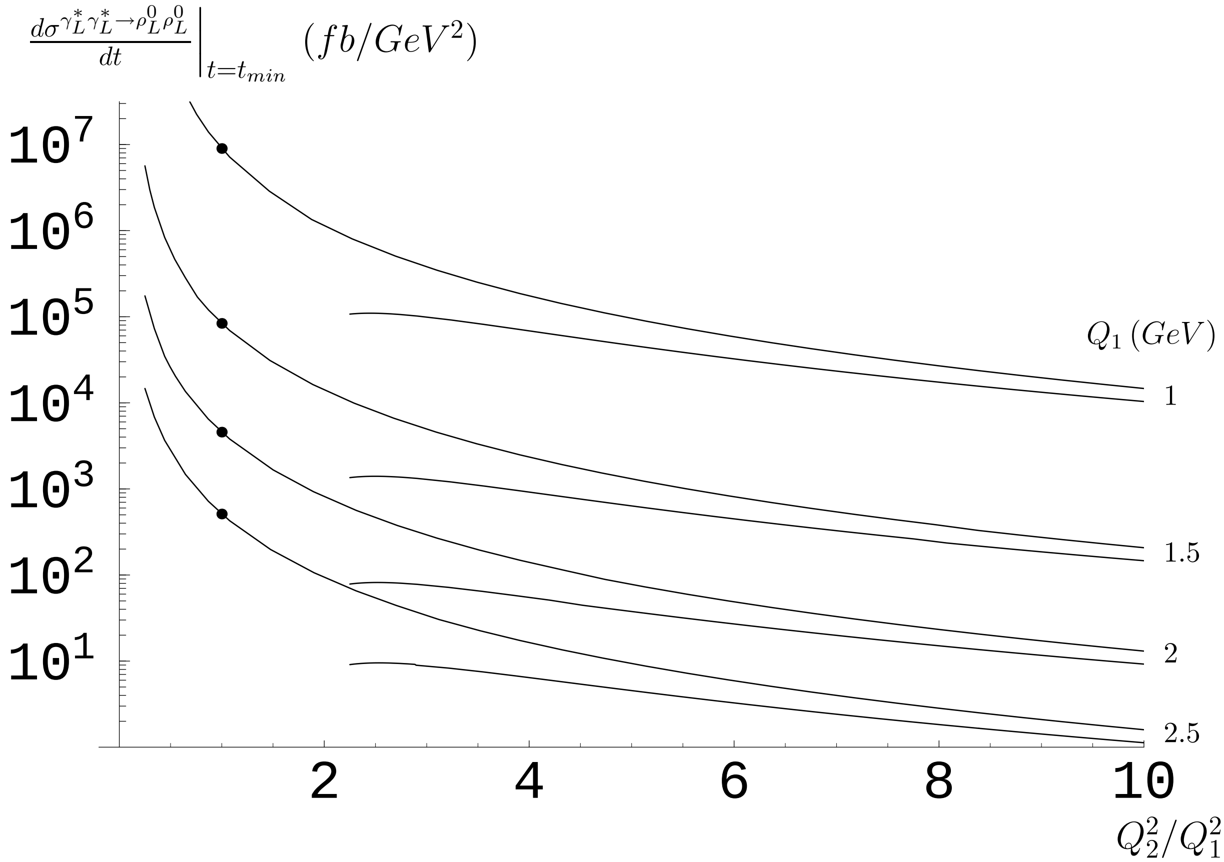

Figure 3: Differential cross-section for the process $\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0$ at Born order, at the threshold $t = t_{min}$, as a function of Q_2^2/Q_1^2. The dots represents the value of the cross-section at the special point $Q_1 = Q_2$, as given by the analytical formula (4.3). The asymptotical curves are valid for large Q_2^2/Q_1^2, as predicted by the asymptotical form (4.4)

.

transverse $t-$channel momentum, *i.e.* $t = t_{min}$, as a function of the ratio Q_2^2/Q_1^2. Curves are labelled by the values of Q_1. The dots on the curves represent the values of the cross-section at the special point $Q_1 = Q_2$, which obviously correspond to the analytical formula (4.3). The cross-section dramatically decreases when Q_2^2/Q_1^2 increases at fixed Q_1. For comparison, we show for each value of Q_1 the asymptotical curve obtained by combining

Eq.(4.2) with Eq.(3.12). The complete result approaches quickly its asymptotical curve. However, in view of the strong decrease of the differential cross-section with increasing Q_2^2/Q_1^2, the asymptotical result seems to be of little interest for estimating data rates in the most favorable kinematics.

The t-dependence of the differential cross-section $d\sigma/dt$ is displayed in Fig. 4 for various values of $Q = Q_1 = Q_2$.

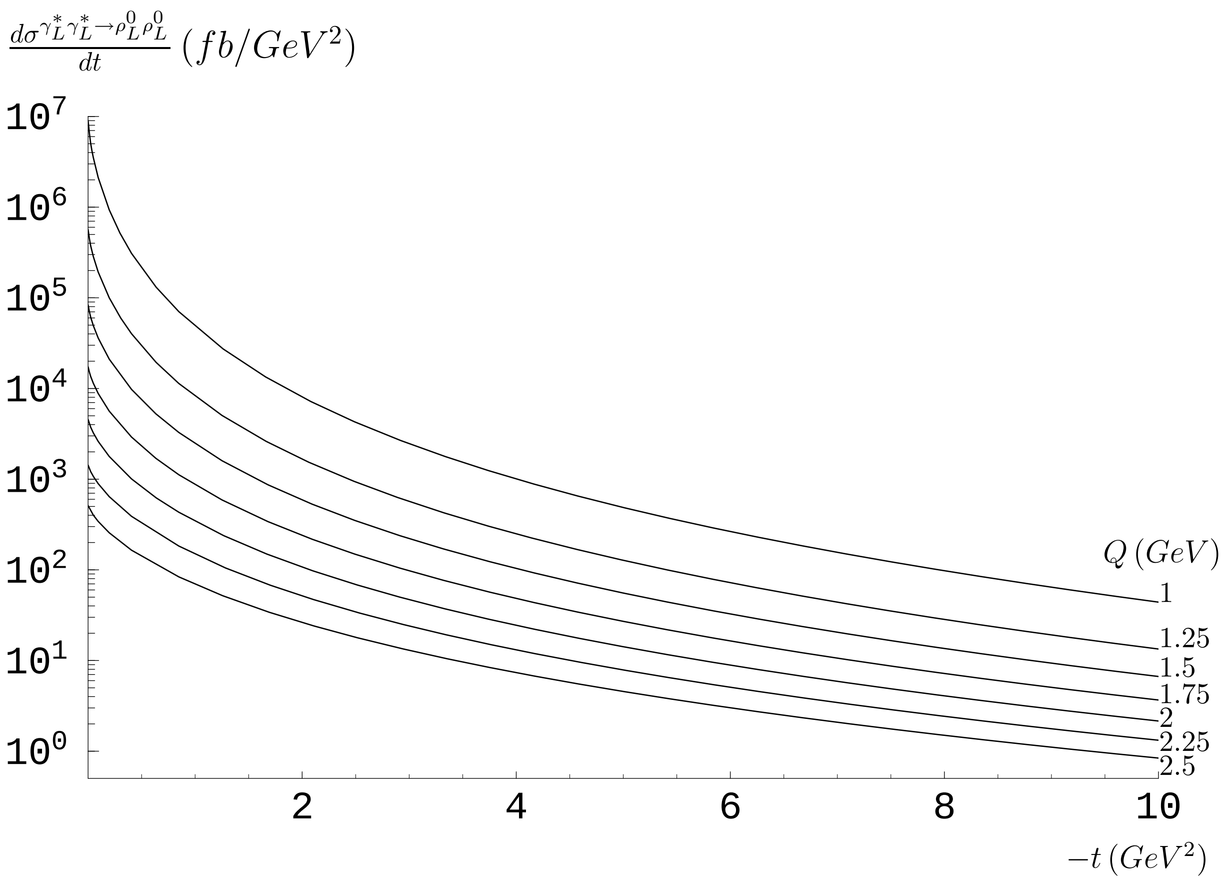

Figure 4: Differential cross-section for the process $\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0$ at Born order, as a function of t, for different values of $Q = Q_1 = Q_2$.

As anticipated, the cross-section is strongly peaked in the forward direction. This fact is less dangerous than for the real photon case since the virtual photon is not in the direction of the beam. However, the differential cross-section seems to be sufficient for the $t-$dependence to be measured up to a few GeV^2. The comparison of the curves on Fig.3 for $t = t_{min}$ with those on Fig.4 leads to the conclusion that the phenomenological predictions obtained in the forward case will practically dictate the general trends of integrated cross-sections.

Figure 5 shows the integrated over t cross-section as a function of $Q^2 = Q_1^2 = Q_2^2$. The magnitude of the cross-section seems to be sufficient for a detailed study to be performed at the linear collider presently under study. Note that we did not multiply by the virtual photon fluxes, which would amplify the dominance of smaller Q^2. However, triggering

efficiency often increases substantially with Q^2 [4]. At this level of calculation there is no s-dependence of the cross-section. It will appear after taking into account of BFKL evolution.

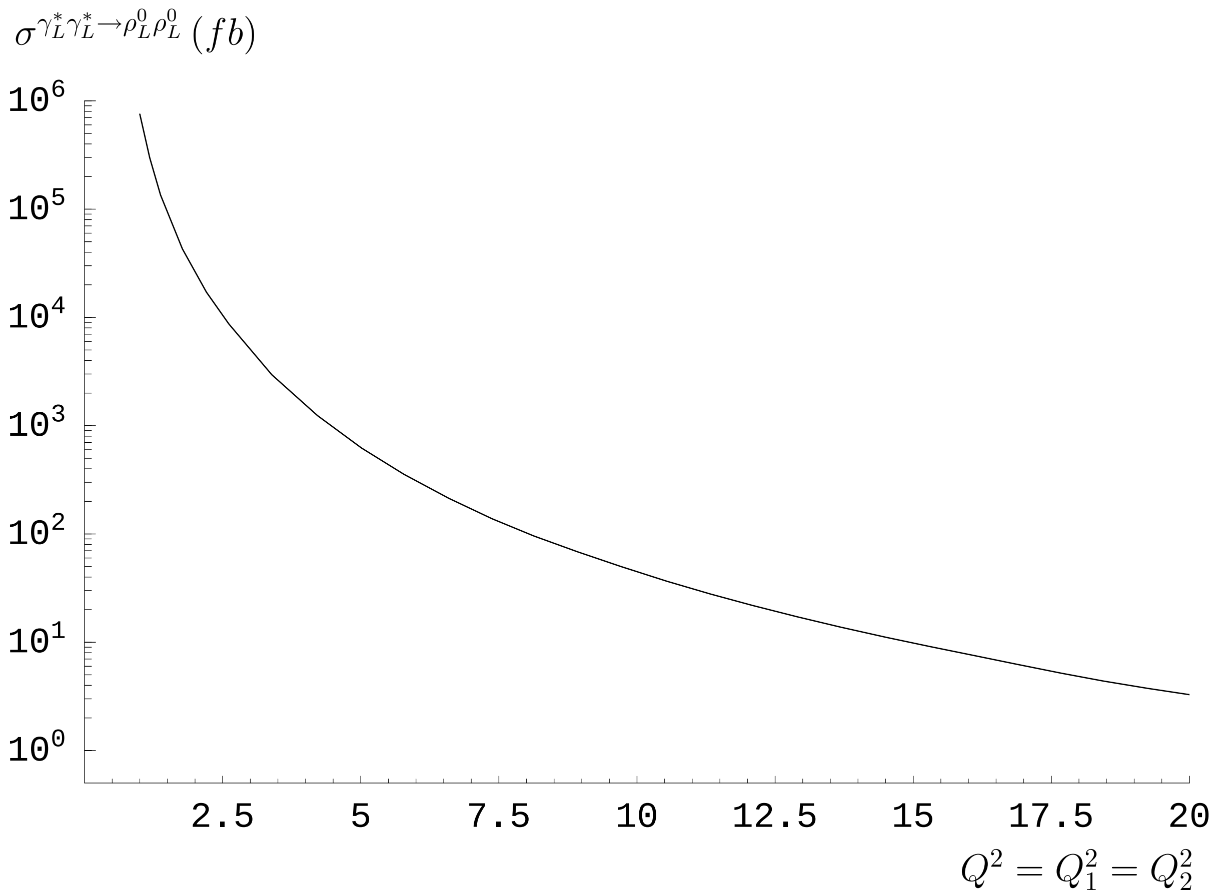

Figure 5: The integrated cross-section for the process $\gamma_L^*\gamma_L^* \to \rho_L^0 \rho_L^0$ at Born order as a function of $Q_1^2 = Q_2^2$.

5 Conclusion

This Born order study shows that the process $\gamma^*(Q_1^2)\gamma^*(Q_2^2) \to \rho_L^0 \rho_L^0$ can be measured at foreseen $e^+ - e^-$−colliders for Q_1^2, Q_2^2 up to a few GeV^2. Indeed, a nominal integrated luminosity of 100fb^{-1} should yield thousands of events per year, with $Q^2 \gtrsim 1$ GeV^2. This would open a new domain of investigation for diffractive processes in which practically all ingredients of the scattering amplitude are under control within a pertubative approach.

In the near future we expect to include the transversely polarized photon contribution, which should slightly enhance the non forward amplitude (this amplitude obviously vanishes at $t = t_{min}$). We also intend to incorporate the effect of BFKL evolution, since resummation effects are expected to give a net and visible enhancement of the cross section.

Finally, let us note that the elusive Odderon may also be looked for in $\gamma^*\gamma^*$ exclusive reactions [16], and that one may use the strategy developed in Ref.[17] to find it through its interference with Pomeron exchange which gives rise to charge asymmetries in $\gamma^*\gamma^* -> \pi\pi\pi\pi$.

NOTE ADDED : After this paper has been accepted, two works [21] improve our analysis by taking into account of higher order effects.

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A Appendices

A.1 Computation of the amplitude at $t = t_{min}$

In this appendix we will prove the result (4.2) for the production amplitude at $|t|_{min} = Q_1^2 Q_2^2/s$, still neglecting the ρ-mass. The amplitude (3.10) can be written as

$$\mathcal{M} = is\, \frac{N_c^2-1}{N_c^2}\, \alpha_s^2\, \alpha_{em}\, \alpha(k_1)\, \beta(k_2)\, f_\rho^2\, \frac{288\pi^2}{Q_1 Q_2}\, K\,, \tag{A.1}$$

where

$$K = \int_0^1 dz_1 \int_0^1 dz_2 \int_0^\infty dk^2 \frac{z_1\,\bar{z}_1 z_2\,\bar{z}_2}{(k^2 + z_1\,\bar{z}_1 Q_1^2)(k^2 + z_2\,\bar{z}_2 Q_2^2)}\,, \tag{A.2}$$

which reduces to

$$K = \int_0^1 dz_1 \int_0^1 dz_2\, \frac{z_1\,\bar{z}_1 z_2\,\bar{z}_2}{z_1\,\bar{z}_1 Q_1^2 - z_2\,\bar{z}_2 Q_2^2}\, \ln \frac{z_1\,\bar{z}_1 Q_1^2}{z_2\,\bar{z}_2 Q_2^2}\,. \tag{A.3}$$

In the following, we denote $R = Q_1/Q_2$. We first reduce the computation of K to a one dimensional integral evaluation. Performing the change of variables $x_1 = 4\, z_1\, \bar{z}_1$ and $x_2 = 4\, z_2\, \bar{z}_2$, K reads

$$K = \frac{1}{4^2 Q_1^2} \int_0^1 \frac{dx_1}{\sqrt{1-x_1}}\, \frac{dx_2\, x_2}{\sqrt{1-x_2}} \left(1 - \frac{1}{1 - R^2\, x_1/x_2}\right) \ln\left(R^2\, \frac{x_1}{x_2}\right)\,. \tag{A.4}$$

After replacing the variable x_1 by $x = x_1/x_2$, K reads

$$K = \frac{1}{4^2 Q_1^2} \int_0^1 \frac{dx_2\, x_2^2}{\sqrt{1-x_2}} \int_0^{1/x_2} \frac{dx}{\sqrt{1 - x\, x_2}} \left(1 - \frac{1}{1-R^2 x}\right) \ln(R^2 x)\,. \tag{A.5}$$

One can now split the integration domain $x_2 \in [0,1] \times x \in [0, 1/x_2]$ as $x \in [0,1] \times x_2 \in [0,1]$ and $x \in [1, \infty] \times x_2 \in [0, 1/x]$. The integral corresponding to the second domain is identical to the first one after exchanging Q_1 and Q_2. Performing the integration over x_2 in the first domain, one gets for K

$$K = \frac{1}{8\,4^2 Q_1^2} \int_0^1 \frac{dx}{x^{5/2}} \left[-6\sqrt{x}\,(1 + x) + (3 + 2\,x + 3x^2)(2\ln(1+\sqrt{x})\ln(1 - x))\right]$$
$$\times \left(1 - \frac{1}{1 - R^2 x}\right) \ln(R^2 x) + (1 \leftrightarrow 2)\,. \tag{A.6}$$

This one dimensional integral, after the change of variable $t = \sqrt{x}$ can finally be reexpressed as

$$K = -\frac{1}{64\,Q_1\,Q_2} \int_0^1 dt \left\{ \frac{6\,R}{t^2}(-2\,t + \ln(1+t) - \ln(1-t))\ln(R\,t) - 6\,(1 + R^2) \right.$$
$$\times \left(\frac{1}{1 - R\,t} - \frac{1}{1 + R\,t}\right) \ln(R\,t) - \frac{6}{R}\ln(1+t)\ln(R\,t) + \frac{6}{R}\ln(1-t)\ln(R\,t)$$
$$+ R\left(3\,R^2 + 2 + \frac{3}{R^2}\right)\left(\frac{\ln(1+t)\ln(R\,t)}{1 - R\,t} + \frac{\ln(1+t)\ln(R\,t)}{1 + R\,t} - \frac{\ln(1-t)\ln(R\,t)}{1 - R\,t}\right.$$
$$\left.\left. - \frac{\ln(1-t)\ln(R\,t)}{1 + R\,t}\right)\right\} + \left(R \leftrightarrow \frac{1}{R}\right)\,. \tag{A.7}$$

The integrals arising from the two first lines of the previous expression, supplemented by the corresponding $R \to 1/R$ contribution, are easily computed by integration by parts through logarithmic and polylogarithmic function Li_2. Using Landen relation for Li_2 (see Chap.1 of Ref.[18]), this simplifies into

$$A = -\frac{1}{64\,Q_1\,Q_2}\left[6\left(R + \frac{1}{R}\right)\ln^2 R + 12\left(R - \frac{1}{R}\right)\ln R + 12\left(R + \frac{1}{R}\right)\right] \tag{A.8}$$

The two last lines of Eq.(A.7) (denoted B in the following) contain terms of the generic form

$$\int \frac{\ln(a + bx)\ln(c + ex)}{f + gx}\,g\,dx\,, \tag{A.9}$$

which can be reduced to the standard form (see Chap.8 of Ref.[18])

$$\int \ln(1-y)\ln(1-cy)\frac{dy}{y}\,. \tag{A.10}$$

This last integral is evaluated in Chap.8 of Ref.[18], for a restricted domain in c. An analytic continuation of such a result for the whole complex domain in c can also be performed [19]. Let us first define

$$\varphi(\alpha) = \arg(e^{i\alpha}) - \alpha \tag{A.11}$$

for any *real* alpha. With this definition, $\frac{\phi(\alpha)}{2\pi}$ is a winding number which counts the number of turns one has to make around 0 in order to bring back α to a value inside the interval $]-\pi,\pi]$. Then, the results for the integral (A.10) reads, for x real

$$\begin{aligned}
\int_0^x \ln(1-y)\,\ln(1-cy)\frac{dy}{y} \;&=\; \mathrm{Li}_3\left(\frac{1-c\,x}{1-x}\right)+\mathrm{Li}_3\left(\frac{1}{c}\right)+\mathrm{Li}_3(1)-\mathrm{Li}_3(1-c\,x)\\
&-\mathrm{Li}_3(1-x)-\mathrm{Li}_3\left(\frac{1-c\,x}{c(1-x)}\right)+\ln(1-x)\,\mathrm{Li}_2(1-c\,x)-\ln(1-c\,x)\,\mathrm{Li}_2(x)\\
&+(\ln(1-c\,x)-\ln(1-x))\,\mathrm{Li}_2\left(\frac{1}{c}\right)+\frac{\pi^2}{6}\ln(1-x)+\frac{1}{2}\ln c\,\ln^2(1-x)\\
&-i\,\pi\,c_1\,\ln^2 c+i\,2\,\pi\,c_1\,\ln c\,\ln(1-c\,x)-i\,2\,\pi\,c_1\,\ln c\,\ln(1-\,x)-4\,\pi^2 c_2\,c_3\,\ln c\\
&+i\,2\,\pi\,(c_1-c_4-c_5)\ln(1-x)\,\ln(1-c\,x)\;-\;4\,\pi^2 c_2\,c_3\,\ln(1-x)\;+\;4\,\pi^2 c_2\,c_3\,\ln(1-c\,x)\\
&-\pi\,(c_1-c_4-c_5)\,i\,\ln^2(1-c\,x)\;+\;i\,\pi\,(-c_1+c_5)\,\ln^2(1-x)+i\,4\,\pi^3\,c_2\,c_3\,,
\end{aligned}\tag{A.12}$$

where

$$\begin{aligned}
c_1 \;&=\; \frac{1}{2\pi}\,\varphi\,(\arg(c-1)-\arg(c)-\arg(1-x))\,,\quad c_2\;=\;\frac{1}{2\pi}\,\varphi\,(-\arg(c))\\
c_3 \;&=\; \frac{1}{2\pi}\,\varphi\,(-\arg(1-x))\,,\quad c_4\;=\;\frac{1}{2\pi}\,\varphi\,(\arg(c)+\arg(x))\\
\text{and}\quad c_5 \;&=\; \frac{1}{2\pi}\,\varphi\,(\arg(x)+\arg(c-1)-\arg(1-x))\,.
\end{aligned}\tag{A.13}$$

c_1, c_2, c_3 and c_4 take either 0 or +1 values while c_5 can take -1, 0 or 1 values. From this formula one can get an analytical form for B. The obtained result is very lenghtly and contains a large bunch of ln, Li_2 and Li_3 of various rational functions of R. A first simplification occurs when using the Landen relation for Li_3

$$\mathrm{Li}_3(x)=\mathrm{Li}_3\left(\frac{1}{x}\right)-\frac{\pi^2}{6}\ln(-x)-\frac{1}{6}\ln^3(-x)+\frac{i}{2}\varphi\,(\arg(1-x)-\arg(-x))\,\ln^2 x\,,\tag{A.14}$$

(see Chap.6 of Ref. [18]), which effectively enables one to get combinations of Li_3 of the only arguments R and $-R$.

Then, one reduces the terms containing Li_2 as combinations of Li_2 of the only arguments R and $-R$. This is possible after using Euler relations for Li_2, which reads

$$\mathrm{Li}_2(z)+\mathrm{Li}_2(1-z)=\frac{\pi^2}{6}-\ln z\,\ln(1-z)\,,\tag{A.15}$$

and

$$\mathrm{Li}_2(z)+\mathrm{Li}_2\left(\frac{1}{z}\right)=\frac{\pi^2}{3}-\frac{1}{2}\ln^2 z\,+i\,(\arg(z-1)-\arg(1-z))\ln z\,,\tag{A.16}$$

as well as Landen relation

$$\mathrm{Li}_2(z)+\mathrm{Li}_2\left(\frac{-z}{1-z}\right)=-\frac{1}{2}\ln^2(1-z)\,+i\,\varphi\,(-\arg(1-z))\,(\ln(1-z)-\ln z-i\,\pi)\,.\tag{A.17}$$

The most non trivial transformation is based on Hill formula (which enables one to expand the double variable function $\mathrm{Li}_2(x\,y)$) continuated in the whole complex plane (see [19] for details),

$$\begin{aligned}\mathrm{Li}_2(x\,y) \;=\; &\mathrm{Li}_2(x) + \mathrm{Li}_2(y) - \mathrm{Li}_2\left[\frac{x(1-y)}{1-xy}\right] - \mathrm{Li}_2\left[\frac{y(1-x)}{1-xy}\right] - \ln\left[\frac{1-x}{1-xy}\right]\ln\left[\frac{1-y}{1-xy}\right]\\ &+ i\,\varphi\,(\arg(1-y) - \arg(1-xy))\left\{\ln\left[\frac{1-x}{1-xy}\right] - \ln\left[\frac{y(1-x)}{1-xy}\right]\right\}\\ &+ i\,\varphi\,(\arg(1-x) - \arg(1-xy))\left\{\ln\left[\frac{1-y}{1-xy}\right] - \ln\left[\frac{x(1-y)}{1-xy}\right]\right\}\\ &+ \varphi\,(\arg(1-y) - \arg(1-xy))\,\varphi\,(\arg(1-x) - \arg(1-xy))\,. \end{aligned} \tag{A.18}$$

After a long succession of painful simplifications, one finally gets

$$\begin{aligned} B \;=\; &-\frac{1}{64\,Q_1\,Q_2}\left(3\,R^2 + \,2\, + \frac{3}{R^2}\right)\left(\ln(1-R)\,\ln^2 R - \ln(R+1)\,\ln^2 R - 2\,\mathrm{Li}_2(-R)\,\ln R\right.\\ &\left.+\,2\,\mathrm{Li}_2(R)\,\ln R \,+\, 2\,\mathrm{Li}_3(-R) \,-\, 2\,\mathrm{Li}_3(R)\right)\,. \end{aligned} \tag{A.19}$$

Using equations (A.8) and (A.19) in order to express $K = A + B$ and using Eq.(A.1), one proves the result (4.2).

The limits $R \gg 1$ and $R \ll 1$ can be easily extracted using the asymptotical formulae for large argument of Li_2 and Li_3. In the case of Li_2 they read

$$\mathrm{Li}_2(x) \sim -\frac{1}{2}\ln^2 x + i(arg(x) - arg(-x))\ln x + \frac{\pi^2}{3} - \frac{1}{x} - \frac{1}{4x^2} - \frac{1}{9x^3} + \mathrm{o}\left(\frac{1}{x^3}\right)\ , \tag{A.20}$$

which reduces in the case of interest here to

$$\mathrm{Li}_2(x) \sim -\frac{1}{2}\ln^2 x - i\,\pi\,\ln x + \frac{\pi^2}{3} - \frac{1}{x} - \frac{1}{4x^2} - \frac{1}{9x^3} + \mathrm{o}\left(\frac{1}{x^3}\right) \quad \text{for} \quad x \to +\infty\ , \tag{A.21}$$

$$\mathrm{Li}_2(x) \sim -\frac{1}{2}\ln^2 x + i\,\pi\,\ln x + \frac{\pi^2}{3} - \frac{1}{x} - \frac{1}{4x^2} - \frac{1}{9x^3} + \mathrm{o}\left(\frac{1}{x^3}\right) \quad \text{for} \quad x \to -\infty\ . \tag{A.22}$$

Similarly for Li_3 one gets from the functional relation (A.14) the asymptotic expansion

$$\mathrm{Li}_3(x) \sim -\frac{1}{6}\ln^3(-x) - \frac{\pi^2}{6}\ln(-x) \,+\, \frac{1}{x} + \frac{1}{8x^2} + \frac{1}{27x^3} + \mathrm{o}\left(\frac{1}{x^3}\right) \quad \text{for} \quad x \to \infty\ , \tag{A.23}$$

which reduces to

$$\mathrm{Li}_3(x) \sim -\frac{1}{6}\ln^3 x - i\frac{\pi}{2}\ln^2 x + \frac{\pi^2}{3}\ln x + \frac{1}{x} + \frac{1}{8x^2} + \frac{1}{27x^3} + \mathrm{o}\left(\frac{1}{x^3}\right) \quad \text{for} \quad x \to +\infty\ , \tag{A.24}$$

and

$$\mathrm{Li}_3(x) \sim -\frac{1}{6}\ln^3 x + i\frac{\pi}{2}\ln^2 x + \frac{\pi^2}{3}\ln x + \frac{1}{x} + \frac{1}{8x^2} + \frac{1}{27x^3} + \mathrm{o}\left(\frac{1}{x^3}\right) \quad \text{for} \quad x \to -\infty\ . \tag{A.25}$$

These asymptotical formulae immediately lead to

$$\mathcal{M}_{t_{\min}} \sim +is\frac{N_c^2-1}{N_c^2}\,\alpha_s^2\,\alpha_{em}\,\alpha(k_1)\,\beta(k_2)\,f_\rho^2\,\frac{96\pi^2}{Q_1^2\,Q_2^2}\left(\frac{\ln R}{R} - \frac{1}{6R}\right)\,. \tag{A.26}$$

A.2 Computation of the amplitude at $t = t_{min}$ in the partonic approach

In this appendix we rederive the asymptotic result (A.26) in a way which corresponds to the usual parton collinear limit. In that limit, this simple results agrees with the fact that each loop of t-channel gluons gives rise to at most one logarithmic term. The logarithmic term corresponds to the leading logarithm approximation (LLA), while the constant term goes beyond this approximation. Let us show that this result can be easily obtained from the representation (3.7). Indeed, in the partonic approach, $k_\perp^2$ is neglected with respect to any scale of the order of Q_1^2, and symmetrically any scale of the order of Q_2^2 is neglected with respect to $k_\perp^2$. It is clear that in such an approximation, one immediately recovers the dominant contribution $\frac{\ln R}{R}$ of (A.26). The question arises how to define a precise prescription in order to get the full leading twist expression (A.26). It turns out that the integral (A.2), provides the proper result

$$K \sim \frac{1}{3Q_1\,Q_2}\left(\frac{\ln R}{R} - \frac{1}{6R}\right)\,, \tag{A.27}$$

after expanding the integrand $\mathcal{K}$ at leading order, namely

$$\mathcal{K} \sim \frac{1}{k^2 Q_1^2 z_1\,\bar{z}_1}\,, \tag{A.28}$$

and then integrating the result from $z_2\,\bar{z}_2/R$ to $R\,z_1\,\bar{z}_1$. Let us justify this prescription. We rewrite the integral K of Eq.(A.2) as

$$K = \frac{4}{Q_1 Q_2}\int_0^{\frac{1}{2}} dz_1 \int_0^{\frac{1}{2}} dz_2 \int_0^{\infty} du \frac{z_1\,\bar{z}_1 z_2\,\bar{z}_2}{(u + R\,z_1\,\bar{z}_1)(u + z_2\,\bar{z}_2/R)}\,, \tag{A.29}$$

and separate the $u = k^2/(Q_1 Q_2)$ integration as

$$\int_0^{\infty} du = \int_{\beta R z_1 \bar{z}_1}^{\infty} du + \int_{\alpha\frac{z_2\bar{z}_2}{R}}^{\beta R z_1 \bar{z}_1} du + \int_0^{\alpha\frac{z_2\bar{z}_2}{R}} du \tag{A.30}$$

where the parameters α and β are arbitrary. In the large R limit, $\beta R z_1\bar{z}_1 < \alpha\frac{z_2\bar{z}_2}{R}$ for $z_1 < \frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}$. Let us perform a systematic expansion of K in the limit where α and β satisfy $R \gg \alpha \gg 1$ and $R \gg 1/\beta \gg 1$. We organize the expansion in such a form that the large R limit is taken first (which means the dominant twist approximation), and only then the large α and small β limit are taken. Decompose $K = K_1 + K_2 + K_3 + K_4 + K_5 + K_6$, where

$$K_1 = \frac{4}{Q_1 Q_2}\int_0^{\frac{1}{2}} dz_2 \int_0^{\frac{\alpha}{\beta}\frac{z_2\bar{z}_2}{R^2}} dz_1 \int_{\alpha\frac{z_2\bar{z}_2}{R}}^{\infty} du \frac{z_1\,\bar{z}_1 z_2\,\bar{z}_2}{(u + R\,z_1\,\bar{z}_1)(u + z_2\,\bar{z}_2/R)} \tag{A.31}$$

$$K_2 = \frac{4}{Q_1 Q_2} \int_0^{\frac{1}{2}} dz_2 \int_0^{\frac{\alpha}{\beta}\frac{z_2 \bar{z}_2}{R^2}} dz_1 \int_{\beta R\, z_1\, \bar{z}_1}^{\alpha\, \frac{z_2 \bar{z}_2}{R}} du \frac{z_1\, \bar{z}_1 z_2\, \bar{z}_2}{(u + R\, z_1\, \bar{z}_1)(u + z_2\, \bar{z}_2/R)} \tag{A.32}$$

$$K_3 = \frac{4}{Q_1 Q_2} \int_0^{\frac{1}{2}} dz_2 \int_0^{\frac{\alpha}{\beta}\frac{z_2 \bar{z}_2}{R^2}} dz_1 \int_0^{\beta R\, z_1\, \bar{z}_1} du \frac{z_1\, \bar{z}_1 z_2\, \bar{z}_2}{(u + R\, z_1\, \bar{z}_1)(u + z_2\, \bar{z}_2/R)} \tag{A.33}$$

$$K_4 = \frac{4}{Q_1 Q_2} \int_0^{\frac{1}{2}} dz_2 \int_{\frac{\alpha}{\beta}\frac{z_2 \bar{z}_2}{R^2}}^{\frac{1}{2}} dz_1 \int_{\beta R\, z_1 \bar{z}_1}^{\infty} du \frac{z_1\, \bar{z}_1 z_2\, \bar{z}_2}{(u + R\, z_1\, \bar{z}_1)(u + z_2\, \bar{z}_2/R)} \tag{A.34}$$

$$K_5 = \frac{4}{Q_1 Q_2} \int_0^{\frac{1}{2}} dz_2 \int_{\frac{\alpha}{\beta}\frac{z_2 \bar{z}_2}{R^2}}^{\frac{1}{2}} dz_1 \int_{\alpha \frac{z_2 \bar{z}_2}{R}}^{\beta R\, z_1\, \bar{z}_1} du \frac{z_1\, \bar{z}_1 z_2\, \bar{z}_2}{(u + R\, z_1\, \bar{z}_1)(u + z_2\, \bar{z}_2/R)} \tag{A.35}$$

$$K_6 = \frac{4}{Q_1 Q_2} \int_0^{\frac{1}{2}} dz_2 \int_{\frac{\alpha}{\beta}\frac{z_2 \bar{z}_2}{R^2}}^{\frac{1}{2}} dz_1 \int_0^{\alpha \frac{z_2 \bar{z}_2}{R}} du \frac{z_1\, \bar{z}_1 z_2\, \bar{z}_2}{(u + R\, z_1\, \bar{z}_1)(u + z_2\, \bar{z}_2/R)}\,. \tag{A.36}$$

The integrals K_1, K_2 and K_3 are of order $1/R^3$ and can thus be neglected at leading twist. It corresponds to the absence of end-point singularities in z variables and means that one could safely replace the lower bound of z_1 integration in K_4, K_5 and K_6 by 0. Let us focus now on the integral K_5. The integration on u runs from $\alpha\, z_2 \bar{z}_2/R$ to $\beta\, R\, z_1\, \bar{z}_1$. Since $z_1\, \bar{z}_1 \leq 1/4$ and $z_2\, \bar{z}_2 \leq 1/4$, in the limit $R \gg 1$, $\alpha \gg 1$ and $\beta \ll 1$, one can safely expand the integrand of K_5 in powers of $u/(R z_1\, \bar{z}_1)$ and $z_2\, \bar{z}_2/(R\, u)$. At leading twist, only the dominant term has to be kept, and $\mathcal{K}$ can be approximated by (A.28). Integration with respect to u, z_1 and z_2 then leads to

$$K_5 \sim \left(\frac{\ln R}{R} - \frac{1}{6R} + \frac{1}{3R} \ln \frac{\beta}{\alpha} \right)\,. \tag{A.37}$$

The logarithmic contribution $\ln \frac{\beta}{\alpha}$ corresponds to boundary effect and is to be completely compensated by K_4 and K_6, which behaves respectively as $1/R \ln \beta$ and $1/R \ln \alpha$ in the limit $R \gg 1$, $\alpha \gg 1$ and $\beta \ll 1$. This justify the assumption stated at the beginning of this paragraph.

A.3 Integrals

In this appendix we collect all the generic integrals which appear in the computation of the Born amplitude. In addition to the integral I_2 defined in Eq.(4.7), we introduce the two following integrals with effective masses

$$I_{2m} = \int \frac{d^d \underline{k}}{\underline{k}^2((\underline{k} - \underline{p})^2 + m^2)}\,, \tag{A.38}$$

and

$$J_{2m_a m_b} = \int \frac{d^d \underline{k}}{((\underline{k}-\underline{a})^2+m_a^2)((\underline{k}-\underline{b})^2+m_b^2)}\,, \tag{A.39}$$

Let us first consider the case of the integral I_{2m}. Using Feynman parametrization, one easily get

$$I_{2m} = \pi^{1+\epsilon}\Gamma(1-\epsilon)(\underline{p}^2+m^2)^{-1+\epsilon}\int\limits_0^1 d\alpha\alpha^{\epsilon-1}\left(1-\alpha\frac{\underline{p}^2}{\underline{p}^2+m^2}\right)^{\epsilon-1}\,. \tag{A.40}$$

In the massless case, one immediately obtains

$$I_2 = \frac{2\pi}{\underline{p}^2\epsilon}(1+\epsilon(\ln(\pi\underline{p}^2)-\Psi(1)))\,. \tag{A.41}$$

For the non zero mass case, the α integration leads to the hypergeometric function ${}_2F_1$,

$$I_{2m} = \pi^{1+\epsilon}\frac{\Gamma(1-\epsilon)}{\epsilon(\underline{p}^2+m^2)^{1-\epsilon}}{}_2F_1\left(1-\epsilon,\epsilon,1+\epsilon;\frac{\underline{p}^2}{\underline{p}^2+m^2}\right)\,. \tag{A.42}$$

After performing the Euler transformation $z \to z/(z-1)$ in the argument of the hypergeometric function and then expanding the result in power of ϵ, one gets

$$I_{2m} = \frac{\pi}{\epsilon(\underline{p}^2+m^2)}(1+\epsilon(\ln\pi-\Psi(1)-\ln m^2+2\ln(\underline{p}^2+m^2)))\,. \tag{A.43}$$

Let us now turn to the more general case where the propagators contain two (different) masses. In this case dimensional regularization is not necessary, and after straightforward calculations, using Feynman parametrization, one obtains

$$\begin{aligned} J_{2m_a m_b} &= \frac{\pi}{\sqrt{((\underline{a}-\underline{b})^2+(m_a-m_b)^2)((\underline{a}-\underline{b})^2+(m_a+m_b)^2)}} \\ &\times \ln\left|\frac{(\underline{a}-\underline{b})^2+m_a^2+m_b^2+\sqrt{((\underline{a}-\underline{b})^2+(m_a-m_b)^2)((\underline{a}-\underline{b})^2+(m_a+m_b)^2)}}{(\underline{a}-\underline{b})^2+m_a^2+m_b^2-\sqrt{((\underline{a}-\underline{b})^2+(m_a-m_b)^2)((\underline{a}-\underline{b})^2+(m_a+m_b)^2)}}\right|\,. \end{aligned} \tag{A.44}$$

For the purpose of our computation, we will need the previous integral only for the special case where $\underline{a}$ and $\underline{b}$ are collinear.

In that case, one obtains, with $r = |\underline{r}|$,

$$\begin{aligned} J_{2\alpha\beta} &= \int \frac{d^d\underline{k}}{((\underline{k}-\underline{r}a)^2+\alpha^2)((\underline{k}-\underline{r}b)^2+\beta^2)} \\ &= \frac{\pi}{\sqrt{\lambda}}\ln\frac{r^2(a-b)^2+\alpha^2+\beta^2+\sqrt{\lambda}}{r^2(a-b)^2+\alpha^2+\beta^2-\sqrt{\lambda}}\,, \end{aligned}$$

where we introduce the notation

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz\,, \tag{A.45}$$

which enables us to define, for the purpose of our computation,

$$\lambda = \lambda(-r^2(a-b)^2, \alpha^2, \beta^2) = (\alpha^2-\beta^2)^2 + 2(\alpha^2+\beta^2)r^2(a-b)^2 + r^4(a-b)^4 . \quad \text{(A.46)}$$

Let us first consider the integral I_{3m}. We start from the following identity

$$\begin{aligned}
&\int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2}\left(\frac{1}{(\underline{k}-\underline{a})^2+m^2} - \frac{1}{\underline{a}^2+m^2} - \frac{1}{(\underline{p}-\underline{a})^2+m^2} + \frac{1}{(\underline{k}-\underline{p}+\underline{a})^2+m^2}\right) \\
&= -\left(\frac{1}{\underline{a}^2+m^2} + \frac{1}{(\underline{p}-\underline{a})^2+m^2}\right)\int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2} \\
&+ \int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2((\underline{k}-\underline{a})^2+m^2)} + \int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2((\underline{k}-\underline{p}+\underline{a})^2+m^2)} . \qquad \text{(A.47)}
\end{aligned}$$

This identity relates a finite expression on the lhs with a sum of dimensionally regularized integrals. After shifting integration variable in the last integral on rhs of Eq.(A.47), one obtains

$$\begin{aligned}
&\int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2((\underline{k}-\underline{a})^2+m^2)} = \frac{1}{2}\left(\frac{1}{\underline{a}^2+m^2} + \frac{1}{(\underline{p}-\underline{a})^2+m^2}\right)\int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2} \qquad \text{(A.48)} \\
&+\frac{1}{2}\int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2}\left(\frac{1}{(\underline{k}-\underline{a})^2+m^2} - \frac{1}{\underline{a}^2+m^2} - \frac{1}{(\underline{p}-\underline{a})^2+m^2} + \frac{1}{(\underline{k}-\underline{p}+\underline{a})^2+m^2}\right),
\end{aligned}$$

which expresses I_{3m} in terms of the divergent integral I_2 already calculated and of a finite integral whose computation is our next task. The method which we use is the generalization for the massive case, in momentum space, of the technique of calculation of massless two dimensional diagrams in the coordinate space encountered in conformal field theories [20].

The essential point is to perform in two dimensional finite integrals a conformal transformation of type $\underline{l} \to \underline{l}/\underline{l}^2$ on the integration variables and vector parameters, and of type $m^2 \to 1/m^2$ for the dimensionful parameters. This transformation reduces the number of propagators.

Let us illustrate this method in the special case where $\underline{a}$ and $\underline{p}$ are collinear, which is of practical interest for our computation. We will thus focus on the finite integral in rhs of Eq.(A.48), which turns out to be J_{3m}, as defined in Eq.(4.11). The transformation

$$\underline{k} \to \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \to \frac{\underline{R}}{\underline{R}^2}, \quad m \to \frac{1}{M} \qquad \text{(A.49)}$$

reduces the number of propagators and gives

$$\begin{aligned}
J_{3m} &= \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\left(\frac{1}{(\underline{k}-\underline{r}a)^2+m^2} - \frac{1}{\underline{r}^2+m^2} + (a \leftrightarrow \bar{a})\right) \\
&= R^2\int \frac{d^2\underline{K}}{(\underline{K}-\underline{R})^2}\left(\frac{K^2R^2}{(\underline{R}-a\underline{K})^2+\frac{\underline{K}^2\underline{R}^2}{M^2}} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a})\right) . \qquad \text{(A.50)}
\end{aligned}$$

After performing the shift of variable $\underline{K} = \underline{R} + \underline{k}'$ and then finally making the inverse transformation

$$\underline{k}' \to \frac{\underline{k}}{\underline{k}^2}, \quad \underline{R} \to \frac{\underline{r}}{\underline{r}^2}, \quad M \to \frac{1}{m}, \tag{A.51}$$

we end up with

$$J_{3m} = \frac{1}{r^2} \int \frac{d^2\underline{k}}{\underline{k}^2} \left[\frac{(\underline{r}+\underline{k})^2}{(r^2a^2+m^2)\left(\left(\underline{k} - \underline{r}\frac{r^2 a\,\underline{a} - m^2}{r^2\underline{a}^2+m^2}\right)^2 + \frac{m^2 r^4}{(r^2\underline{a}^2+m^2)^2}\right)} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a}) \right] . \tag{A.52}$$

The computation of this integral can now be performed using standard Feynman parameter technique. It results in

$$J_{3m} = \frac{2\pi}{r^2} \left\{ \left(\frac{1}{r^2a^2+m^2} - \frac{1}{r^2\bar{a}^2+m^2} \right) \ln \frac{r^2a^2+m^2}{r^2\bar{a}^2+m^2} \right. \tag{A.53}$$

$$\left. + \left(\frac{1}{r^2a^2+m^2} + \frac{1}{r^2\bar{a}^2+m^2} + \frac{2}{r^2a\bar{a}-m^2} \right) \ln \frac{(r^2a^2+m^2)(r^2\bar{a}^2+m^2)}{m^2r^2} \right\} . \tag{A.54}$$

Let us now focus on the I_{4mm} integral. We start with an analogous identity as those in Eq.(A.47), which leads to

$$\int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2((\underline{k}-\underline{a})^2+m_a^2)(\underline{k}-\underline{b})^2+m_b^2)} \tag{A.55}$$

$$= \frac{1}{2} \left(\frac{1}{(\underline{a}^2+m_a^2)(\underline{b}^2+m_b^2)} + \frac{1}{((\underline{p}-\underline{a})^2+m_a^2)((\underline{p}-\underline{b})^2+m_b^2)} \right) \int \frac{d^d\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2} + \frac{1}{2} J ,$$

where

$$J = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{p})^2} \left(\frac{1}{((\underline{k}-\underline{a})^2+m_a^2)((\underline{k}-\underline{b})^2+m_b^2)} - \frac{1}{(\underline{a}^2+m_a^2)(\underline{b}^2+m_b^2)} \right. \tag{A.56}$$

$$\left. - \frac{1}{((\underline{p}-\underline{a})^2+m_a^2)((\underline{p}-\underline{b})^2+m_b^2)} + \frac{1}{((\underline{k}-\underline{p}+\underline{a})^2+m_a^2)((\underline{k}-\underline{p}+\underline{b})^2+m_b^2)} \right) .$$

This identity express the integral $I_{4m_am_b}$ as a sum of a IR divergent integral which has already been computed in Eq.(A.41) and of a finite integral J, on which we will apply the same trick based on conformal transformations. Once more, although our method can be applied to the previous integral J, we will restrict ourselves to the more simple case where a and b are collinear. It thus means that we need to compute the integral $J_{4\mu_1\mu_2}$ as defined in Eq.(4.12). We apply on $J_{4\mu_1\mu_2}$ three successive transformations accompanied by appropriate changes of variables as described above: conformal transformation, shift of integration variable and inverse conformal transformation. This gives

$$J_{4\mu_1\mu_2} = \frac{1}{\underline{r}^2} \int \frac{d^d\underline{k}}{\underline{k}^2} \tag{A.57}$$

$$\times \left\{ \frac{((\underline{k}-\underline{r})^2)^2}{(r^2\bar{z}_1+\mu_1^2)\left(\left(\underline{k}+\underline{r}\frac{z_1\bar{z}_1 r^2-\mu_1^2}{\bar{z}_1^2 r^2+\mu_1^2}\right)^2+\frac{r^4\mu_1^2}{(r^2 z_1^2+\mu_1^2)^2}\right)(r^2\bar{z}_2+\mu_2^2)\left(\left(\underline{k}+\underline{r}\frac{z_2\bar{z}_2 r^2-\mu_2^2}{\bar{z}_2^2 r^2+\mu_2^2}\right)^2+\frac{r^4\mu_2^2}{(r^2 z_2^2+\mu_2^2)^2}\right)} \right.$$

$$\left. -\frac{1}{(r^2\bar{z}_1+\mu_1^2)(r^2\bar{z}_2+\mu_2^2)} + (z \leftrightarrow \bar{z}) \right\} .$$

In this expression, the integral which remains to be computed has the form

$$J_{3\alpha\beta} = \int \frac{d^2\underline{k}((\underline{k}-\underline{r})^2)^2}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}\,, \tag{A.58}$$

This integral can be rewritten in the form

$$J_{3\alpha\beta} = J_{3\alpha\beta}^{UV} + J_{3\alpha\beta}^{IR} \tag{A.59}$$

where

$$J_{3\alpha\beta}^{UV} = \int \frac{d^d\underline{k}\,\underline{k}^2}{[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]} \tag{A.60}$$

and

$$J_{3\alpha\beta}^{IR} = \int \frac{d^d\underline{k}\,[((\underline{k}-\underline{r})^2)^2-(\underline{k}^2)^2]}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}\,. \tag{A.61}$$

$J_{3\alpha\beta}^{UV}$ is IR finite but UV divergent. On the contrary $J_{3\alpha\beta}^{IR}$ is UV finite but IR divergent. The calculation of $J_{3\alpha\beta}^{UV}$ is standard, although somehow lenghty, and leads to the result

$$J_{3\alpha\beta}^{UV} = \pi \left\{ -\frac{\pi^\epsilon\Gamma(1-\epsilon)}{\epsilon} - \frac{1}{2}\ln(\alpha^2\,\beta^2) + \frac{\alpha^2-\beta^2}{2r^2(a-b)^2}\ln\frac{\alpha^2}{\beta^2} \right. \tag{A.62}$$

$$-\frac{\sqrt{\lambda}}{2r^2(a-b)^2}\ln\frac{r^2(a-b)^2+\alpha^2+\beta^2+\sqrt{\lambda}}{r^2(a-b)^2+\alpha^2+\beta^2-\sqrt{\lambda}}$$

$$-\frac{[\sqrt{\lambda}-\alpha^2+\beta^2-r^2(a^2-b^2)]^2}{4r^2(a-b)^2\sqrt{\lambda}}\ln\frac{\sqrt{\lambda}-r^2(a-b)^2-\alpha^2+\beta^2}{\sqrt{\lambda}+r^2(a-b)^2-\alpha^2+\beta^2}$$

$$\left. -\frac{[\sqrt{\lambda}-\beta^2+\alpha^2-r^2(b^2-a^2)]^2}{4r^2(a-b)^2\sqrt{\lambda}}\ln\frac{\sqrt{\lambda}-r^2(a-b)^2+\alpha^2-\beta^2}{\sqrt{\lambda}+r^2(a-b)^2+\alpha^2-\beta^2} \right\} . \tag{A.63}$$

The evaluation of $J_{3\alpha\beta}^{IR}$ can be simplified by expressing it in the following way:

$$J_{3\alpha\beta}^{IR} = \int d^d\underline{k}\frac{2\underline{r}^2}{[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]} + r^4\int \frac{d^d\underline{k}}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}$$

$$-4r^2\int \frac{d^d\underline{k}\,\underline{k}\cdot\underline{r}}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]} - 4\int \frac{d^d\underline{k}\,\underline{k}\cdot\underline{r}}{[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}$$

$$+4\int \frac{d^d\underline{k}\,(\underline{k}\cdot\underline{r})^2}{\underline{k}^2[(\underline{k}-\underline{r}a)^2+\alpha^2]\,[(\underline{k}-\underline{r}b)^2+\beta^2]}\,. \tag{A.64}$$

Each of these integrals is regular, except the second one, which diverges like

$$J_{3\alpha\beta}^{IR\,div.} \equiv \frac{\pi^{(1+\epsilon)}}{\epsilon\,\Gamma(1+\epsilon)} \frac{r^4}{(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}\,. \tag{A.65}$$

The evaluation of each of these integrals is lenghty but straighforward after using Feynman parametrization.

Since IR and UV divergences cancel when evaluating the integral I defined in Eq.(A.57), we effectively need the finite part of $J_{3\alpha\beta}$, defined as the remaining term after removing the pole in $1/\epsilon$, which sums up the finite part of $J_{3\alpha\beta}^{IR}$ and of $J_{3\alpha\beta}^{UV}$. With this definition, one can express $J_{4\mu_1\mu_2}$ as

$$\begin{aligned} &J_{4\mu_1\mu_2} = \\ &\frac{1}{r^2(r^2\bar{z}_1^2+\mu_1^2)(r^2\bar{z}_2^2+\mu_2^2)} J_{3\alpha\beta}^{finite}\left(\frac{-z_1\bar{z}_1r^2+\mu_1^2}{\bar{z}_1^2r^2+\mu_1^2}, \frac{-z_2\bar{z}_2r^2+\mu_2^2}{\bar{z}_2^2r^2+\mu_2^2}, \frac{r^2\mu_1}{r^2\bar{z}_1^2+\mu_1^2}, \frac{r^2\mu_2}{r^2\bar{z}_2^2+\mu_2^2}, r\right) \\ &+\frac{1}{r^2(r^2z_1^2+\mu_1^2)(r^2z_2^2+\mu_2^2)} J_{3\alpha\beta}^{finite}\left(\frac{-z_1\bar{z}_1r^2+\mu_1^2}{z_1^2r^2+\mu_1^2}, \frac{-z_2\bar{z}_2r^2+\mu_2^2}{z_2^2r^2+\mu_2^2}, \frac{r^2\mu_1}{r^2z_1^2+\mu_1^2}, \frac{r^2\mu_2}{r^2z_2^2+\mu_2^2}, r\right) \end{aligned} \tag{A.66}$$

with

$$\begin{aligned} &J_{3\alpha\beta}^{finite}(a,\,b,\,\alpha,\,\beta,\,r) \\ =\ &\pi\left\{\frac{1}{2\sqrt{\lambda}}\left[-\frac{(\alpha-\beta)^2\,(\alpha+\beta)^2}{(a-b)^2\,r^2} - 4\frac{\alpha^2-\beta^2}{a-b} - 2(\alpha^2+\beta^2) - ((a-b)^2+4(a+b)-12)r^2\right.\right. \\ &\left.-r^2\left(\frac{1}{r^2a^2+\alpha^2}+\frac{1}{r^2b^2+\beta^2}\right)\right]\ln\frac{r^2(a-b)^2+\alpha^2+\beta^2+\sqrt{\lambda}}{r^2(a-b)^2+\alpha^2+\beta^2-\sqrt{\lambda}} \\ &+\frac{1}{\sqrt{\lambda}}\left[r^2(a-b)^2+2(\alpha^2+\beta^2)-2\frac{a\,\beta^2-b\,\alpha^2}{a-b}+2a\,b\,r^2+r^2\frac{\alpha^2-\beta^2+(a^2-b^2)r^2}{b(r^2a^2+\alpha^2)-a(r^2b^2+\beta^2)}\right. \\ &\times\left(\frac{r^2a}{r^2a^2+\alpha^2}+\frac{r^2b}{r^2b^2+\beta^2}-4\right)+2\frac{(\alpha^2-\beta^2)^2+2(\alpha^2+\beta^2)^2\,r^2(a-b)^2+r^4(a-b)^4}{b\,(r^2a^2+\alpha^2)-a\,(r^2b^2+\beta^2)}\frac{1}{a-b} \\ &\left.+\frac{(\alpha^2-\beta^2)^2}{r^2(a-b)^2}\right]\ln\frac{\sqrt{\lambda}+r^2(a-b)^2+\alpha^2-\beta^2}{\sqrt{\lambda}-r^2(a-b)^2+\alpha^2-\beta^2}+\left[-\frac{1}{ab(a-b)}\cdot\frac{a^2\beta^2-b^2\alpha^2+ab\sqrt{\lambda}}{a(r^2b^2+\beta^2)-b(r^2a^2+\alpha^2)}\right. \\ &-\frac{r^2}{2\sqrt{\lambda}}\left(\frac{r^2a}{r^2a^2+\alpha^2}+\frac{r^2b}{r^2b^2+\beta^2}-4\right)\frac{\sqrt{\lambda}+\alpha^2-\beta^2-r^2(b^2-a^2)}{a(r^2b^2+\beta^2)-b(r^2a^2+\alpha^2)}+\frac{2}{a-b}+\frac{a\alpha^2-b\beta^2}{(a-b)\sqrt{\lambda}} \\ &\left.+\frac{(\alpha^2-\beta^2)^2}{2r^2(a-b)^2\sqrt{\lambda}}+\frac{(a^2+b^2)r^2}{2\sqrt{\lambda}}-\frac{a+b}{2(a-b)}\right]\ln\frac{\alpha^2}{\beta^2}+\left[\frac{1}{ab}-\frac{r^4}{2(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}-\frac{1}{2}\right] \\ &\times\ln\frac{\alpha^2\,\beta^2}{r^4}+\frac{1}{a(r^2b^2+\beta^2)-b(r^2a^2+\alpha^2)}\left[r^2\left(\frac{r^2a}{r^2a^2+\alpha^2}+\frac{r^2b}{r^2b^2+\beta^2}-4\right)+\frac{a^2r^2+\alpha^2}{a}\right. \\ &\left.\left.+\frac{b^2r^2+\beta^2}{b}\right]\ln\frac{r^2a^2+\alpha^2}{r^2b^2+\beta^2}-\frac{(r^2a^2+\alpha^2)(r^2b^2+\beta^2)-abr^4}{ab(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}\ln\frac{(r^2a^2+\alpha^2)(r^2b^2+\beta^2)}{r^4}\right\}\,, \end{aligned} \tag{A.67}$$

where appropriate additional $\ln r^2$ terms have been introduced after extracting the finite part of $J_{3\alpha\beta}^{UV}$ and $J_{3\alpha\beta}^{IR}$ in order to write the final result as made of logarithms of dimensionless quantities. This is possible since the final result $J_{4\mu_1\mu_2}$ is UV and IR finite.

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